Inverse Interpolation

Compiled by

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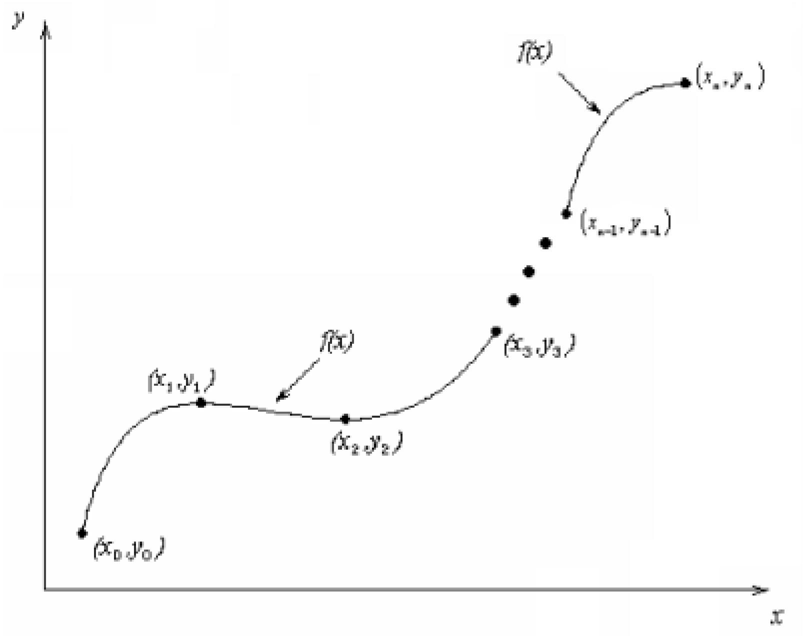
What is Inverse Interpolation ?

- Given (x_0,y_0), (x_1,y_1), …… (x_n,y_n), find the value of 'x' at a value of 'y' that is not given.



Interpolation: Lagrange formula

- We have been finding the value of y corresponding to a certain value of x from a given set of values of x and y by using the Lagrange formula given by

$$y_g=(y_0)(L_0) + (y_1)(L_1) + (y_2)(L_2) +\ldots\ldots\ldots\ldots+(Y_{n-1})(L_{n-1})$$

- $L_0 = (x_g - x_1)\,(x_g - x_2)\ldots\ldots\,(x_g - x_{n-1})/\,(x_0 - x_1\,)\,(x_0 - x_2\,)\,\ldots\ldots\,(x_0 - x_{n-1}\,)$
- $L_1 = (x_g - x_0)\,(x_g - x_2)\ldots\ldots\,(x_g - x_{n-1})/\,(x_1 - x_0\,)\,(x_1 - x_2\,)\,\ldots\ldots\,(x_1 - x_{n-1}\,)$
- $L_2 = (x_g - x_0)\,(x_g - x_1)\ldots\ldots\,(x_g - x_{n-1})/\,(x_2 - x_0\,)\,(x_2 - x_1\,)\,\ldots\ldots\,(x_2- x_{n-1}\,)$
- ………………………………………………………………………………………
- $L_{n-1} = (x_g - x_0)\,(x_g - x_1)\ldots\ldots\,(x_g - x_{n-2})/\,(x_{n-1} - x_0\,)\,(x_{n-1} - x_1)\,\ldots\ldots\,(x_{n-1} - x_{n-2}\,)$

Inverse Interpolation: Lagrange formula

- On the other hand, the process of estimating the value of x for a value of y is called inverse interpolation.

- When the values of y are unequally spaced, Lagrange's method is used

- and when the values of y are equally spaced, Newton's forward difference formula or iterative method can be used.

Inverse Interpolation: Lagrange formula…cont

- Lagrange's formula for inverse interpolation is used when we are required to find the value of x corresponding to a certain value of y from a given set of values of x and y. The formula is as follows
- $x_g = (x_0)(L_0) + (x_1)(L_1) + (x_2)(L_2) + \ldots\ldots\ldots\ldots + (x_{n-1})(L_{n-1})$
- $L_0 = (y_g - y_1)\,(y_g - y_2) \ldots\ldots\, (y_g - y_{n-1}) / \,(y_0 - y_1)\,(y_0 - y_2) \ldots\ldots\, (y_0 - y_{n-1})$
- $L_1 = (y_g - y_0)\,(y_g - y_2) \ldots\ldots\, (y_g - y_{n-1}) / \,(y_1 - y_0)\,(y_1 - y_2) \ldots\ldots\, (y_1 - y_{n-1})$
- $L_2 = (y_g - y_0)\,(y_g - y_1) \ldots\ldots\, (y_g - y_{n-1}) / \,(y_2 - y_0)\,(y_2 - y_1) \ldots\ldots\, (y_2 - y_{n-1})$
- ………………………………………………………………………………………….
- $L_{n-1} = (y_g - y_0)\,(y_g - y_1) \ldots\ldots\, (y_g - y_{n-2}) / \,(y_{n-1} - y_0)\,(y_{n-1} - y_1) \ldots\ldots\, (y_{n-1} - y_{n-2})$

Inverse Interpolation: Lagrange formula…cont

- Example: The following are co-ordinates of a set of points. Find x at y=2 using Lagrange formula

x	0	1	2	3
y	0	1	7	25

- Solution: In this case we have, $y_g = 2$
- $L_0 = (y_g - y_1)(y_g - y_2)(y_g - y_3) / (y_0 - y_1)(y_0 - y_2)(y_0 - y_3)$

  $=(2 - 1)(2 - 7)(2 - 25) / (0 - 1)(0 - 7)(0 - 25)$

  $= -0.6571$

Inverse Interpolation: Lagrange formula…cont

- $L_1 = (y_g - y_0)\ (y_g - y_2)\ (y_g - y_3) /\ (y_1 - y_0)\ (y_1 - y_2)\ (y_1 - y_3)$

  $=(2 - 0)\ (2 - 7)\ (2 - 25) /\ (1 - 0)\ (1 - 7)\ (1 - 25)$

  $= 1.5972$

- $L_2 = (y_g - y_0)\ (y_g - y_1)\ (y_g - y_3) /\ (y_2 - y_0)\ (y_2 - y_1)\ (y_2 - y_3)$

  $=(2 - 0)\ (2 - 1)\ (2 - 25) /\ (7 - 0)\ (7 - 1)\ (7 - 25)$

  $= 0.0608$

The Lagrange Polynomial: Degree n Construction

- $L_3 = (y_g - y_0)\,(y_g - y_1)\,(y_g - y_2) / (y_3 - y_0)\,(y_3 - y_1)\,(y_3 - y_2)$

  $=(2 - 0)\,(2 - 1)\,(2 - 7) / (25 - 0)\,(25 - 1)\,(25 - 7)$

  $= -\,9.2592 \times 10^{-4}$

$x_g = (x_0)(L_0) + (x_1)(L_1) + (x_2)(L_2) + (x_3)(L_3)$

$= (0)(-\,0.6571) + (1)(1.5972) + (2)(0.0608) + (3)(-\,9.2592 \times 10^{-4})$

$x_g = 1.7160$ at $y_g = 2$

Problems on Inverse Interpolation

1. The following table gives the values of x and y.

x	1.2	2.1	2.8	4.1
y	4.2	6.8	9.8	13.4

Find the value of x corresponding to y=12 using suitable method.

Problems on Inverse Interpolation

2. For the given table find the value of x for f(x)=0.390

x	20	25	30	35
f(x)	0.342	0.423	0.5	0.65

Problems on Inverse Interpolation

3. The following data gives the values of y corresponding to certain values of x. Find the value of x when y=167.59789 by applying Lagrange's method.

x	1	2	5	7
y	1	12	117	317

Reference Books

1. Steven C. Chapra, Raymond P. Canale, Numerical Methods for Engineers, 4/e, Tata McGraw Hill Editions

2. Dr. B. S. Garewal, Numerical Methods in Engineering and Science, Khanna Publishers,.

3. Steven C. Chapra, Applied Numerical Methods with MATLAB for Engineers and Scientist, Tata Mc-Graw Hill Publishing Co-Ltd

4. Rao V. Dukkipati, Applied Numerical Methods using Matlab, New Age International Publishers

Reference Books ...Contd

5. Gerald and Wheatley, Applied Numerical Analysis, Pearson Education Asia
6. E. Balagurusamy, Numerical Methods, Tata McGraw Hill
7. P. Thangaraj, Computer Oriented Numerical Methods, PHI
8. S. S. Sastry, Introductory Methods of Numerical Analysis, PHI.

Thank You

Lagrange Method of Interpolation

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What is Interpolation ?

- Given (x_0,y_0), (x_1,y_1), …… (x_n,y_n), find the value of ‘y’ at a value of ‘x’ that is not given.

Lagrange Interpolation

Lagrangian interpolating polynomial is given by

$$f_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

where 'n' in $f_n(x)$ stands for the n^{th} order polynomial that approximates the function $y = f(x)$ given at $(n+1)$ data points as $(x_0, y_0), (x_1, y_1), \ldots\ldots, (x_{n-1}, y_{n-1}), (x_n, y_n)$, and

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

$L_i(x)$ is a weighting function that includes a product of $(n-1)$ terms with terms of $j = i$ omitted.

The Lagrange Polynomial: The Linear Case

The problem of determining a polynomial of degree one that passes through the distinct points

(x_0, y_0) and (x_1, y_1)

is the same as approximating a function f for which

$f(x_0) = y_0$ and $f(x_1) = y_1$

by means of a first-degree polynomial interpolating, or agreeing with, the values of f at the given points.

Using this polynomial for approximation within the interval given by the endpoints is called polynomial interpolation.

The Lagrange Polynomial: The Linear Case …cont

- Define the functions $L_0(x) = (x - x_1)/ (x_0 - x_1)$ and $L_1(x) = (x - x_0) /(x_1 - x_0)$.

- The linear Lagrange interpolating polynomial though (x_0, y_0) and (x_1, y_1) is

- $P(x) = L_0(x)f(x_0) + L_1(x)f(x_1)$

$=((x - x_1)/ (x_0 - x_1)) f(x_0) + ((x - x_0)/ (x_1 - x_0)) f(x_1)$

The Lagrange Polynomial: The Linear Case …cont

- Note that $L_0(x_0) = 1$, $L_0(x_1) = 0$, $L_1(x_0) = 0$, and $L_1(x_1) = 1$,
- which implies that

$$P(x_0) = 1 \cdot f(x_0) + 0 \cdot f(x_1) = f(x_0) = y_0$$

and

$$P(x_1) = 0 \cdot f(x_0) + 1 \cdot f(x_1) = f(x_1) = y_1.$$

- So P is the unique polynomial of degree at most 1 that passes through (x_0, y_0) and (x_1, y_1).

The Lagrange Polynomial: The Linear Case …cont

- Example: Linear Interpolation
- Determine the linear Lagrange interpolating polynomial that passes through the points (2, 4) and (5, 1).
- Solution: In this case we have

$$L_0(x) = (x - 5)/(2 - 5) = -\,(1/3)\,(x - 5) \text{ and}$$

$$L_1(x) = (x - 2)/\,(5 - 2) = (1/3)\,(x - 2),$$

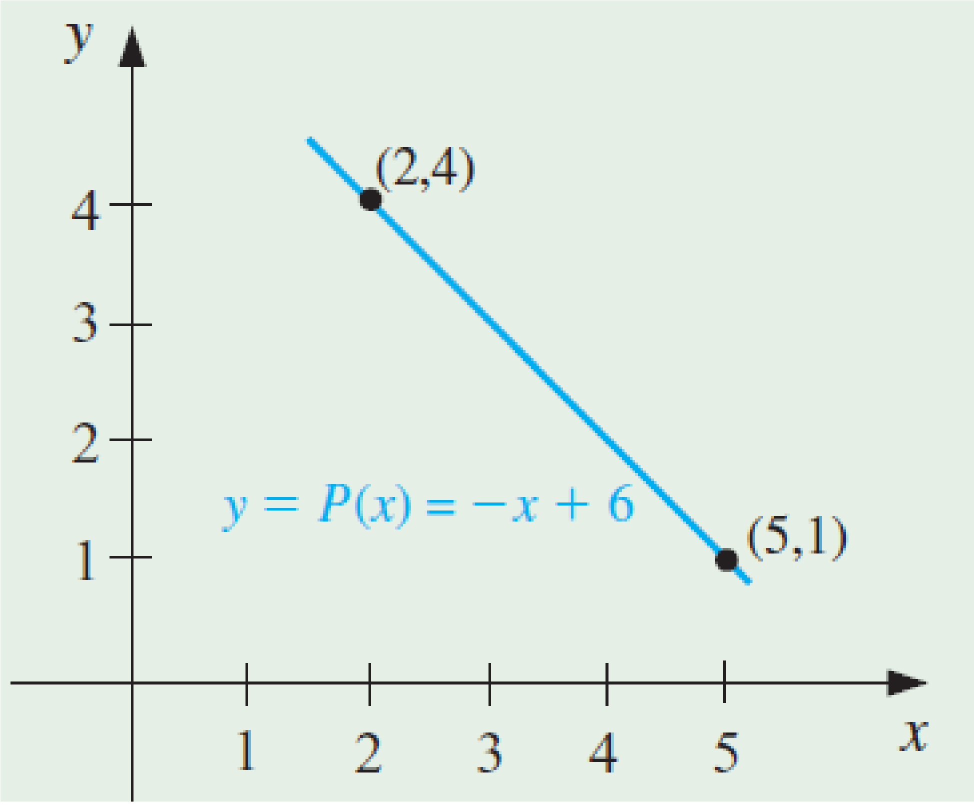
- so

$$P(x) = -\,(1/3)\,(x - 5) * 4 + (1/3)\,(x - 2) *1$$

$$= -(4/3)x+(20/3)+(1/3)x-(2/3)$$

$$P(x) = -x + 6.$$

The Lagrange Polynomial: The Linear Case …cont



- The linear Lagrange interpolating polynomial that passes through the points (2, 4) and (5, 1).

The Lagrange Polynomial: Degree n Construction

- To generalize the concept of linear interpolation, consider the construction of a polynomial of degree at most n that passes through the n + 1 points

$$(x_0, f(x_0)), (x_1, f(x_1)), \ldots, (x_n, f(x_n)).$$

- Constructing the Degree n Polynomial

We first construct, for each $k = 0, 1, \ldots, n$, a function $L_{n,k}(x)$ with the property that $L_{n,k}(x_i) = 0$ when $i \neq k$ and $L_{n,k}(x_k) = 1$.

The Lagrange Polynomial: Degree n Construction ...cont

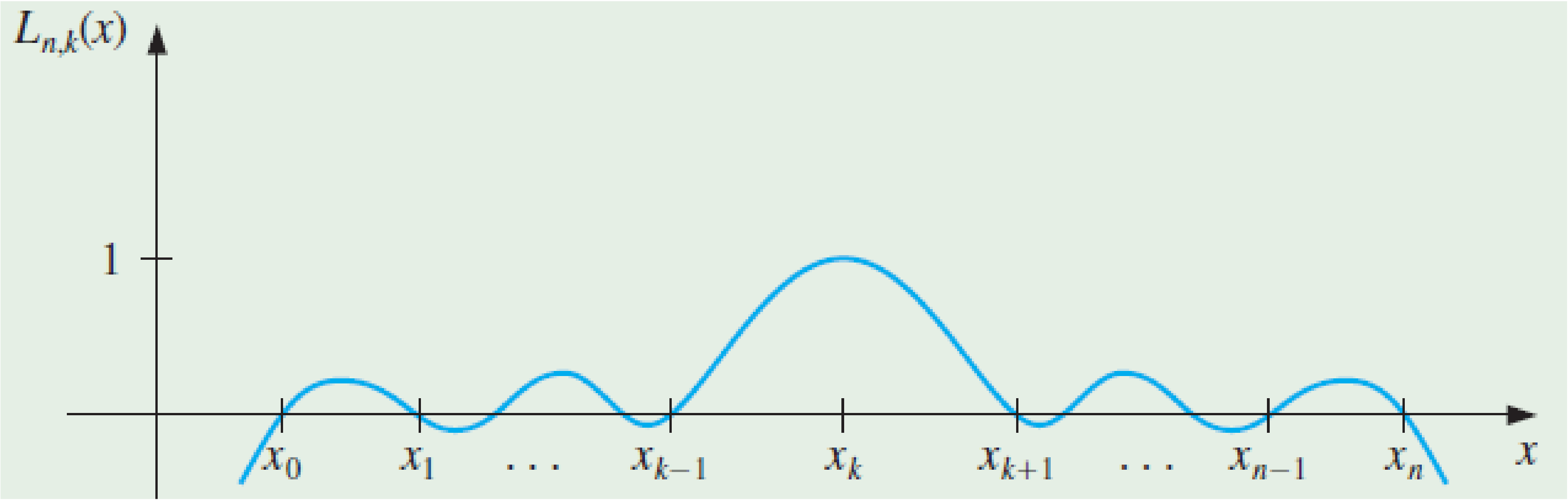
- To satisfy $L_{n,k}(x_i) = 0$ for each $i \neq k$ requires that the numerator of $L_{n,k}(x)$ contain the term

$$(x - x_0)(x - x_1) \cdots (x - x_{k-1})(x - x_{k+1}) \cdots (x - x_n).$$

- To satisfy $L_{n,k}(x_k) = 1$, the denominator of $L_{n,k}(x)$ must be this same term but evaluated at $x = x_k$.
- Thus

$$L_{n,k}(x) = [(x - x_0) \cdots (x - x_{k-1})(x - x_{k+1}) \cdots (x - x_n)] \, / \, [(x_k - x_0) \cdots (x_k - x_{k-1})(x_k - x_{k+1}) \cdots (x_k - x_n)].$$

The Lagrange Polynomial: Degree n Construction ...cont



The Lagrange Polynomial: Degree n Construction …cont

- Example: f (x) = (1/x)

(a) Use the numbers (called nodes) $x_0 = 2$, $x_1 = 2.75$ and $x_2 = 4$ to find the second Lagrange interpolating polynomial for f (x) = 1/x .

(b) Use this polynomial to approximate f (3) = 1/3

The Lagrange Polynomial: Degree n Construction …cont

- Part (a): Solution
- We first determine the coefficient polynomials $L_0(x)$, $L_1(x)$, and $L_2(x)$:
- $L_0(x) = [(x - 2.75)(x - 4)] / [(2 - 2.5)(2 - 4)]$

  $= (2/3)[(x - 2.75)(x - 4)]$
- $L_1(x) = [(x - 2)(x - 4)] / [(2.75 - 2)(2.75 - 4)]$

  $= -(16/15)[(x - 2)(x - 4)]$
- $L_2(x) = [(x - 2)(x - 2.75)] / [(4 - 2)(4 - 2.5)]$

  $= (2/5)[(x - 2)(x - 2.75)]$

The Lagrange Polynomial: Degree n Construction ...cont

- Also, since f (x) = (1/x) :
- f (x_0) = f (2) = 1/2, f (x_1) = f (2.75) = 4/11, f (x_2) = f (4) = ¼
- Therefore, we obtain

$$P(x) = \sum_{k=0}^{2}(f(x_k)L_k(x))$$

= (1/3)[(x – 2.75)(x – 4)]– (64/165)[(x – 2)(x – 4) + (1/10)[(x – 2)(x – 2.75)]

= (1/22)x^2 –(35/88) x + (49/44).

The Lagrange Polynomial: Degree n Construction ...cont

(b) Use this polynomial to approximate $f(3) = 1/3$

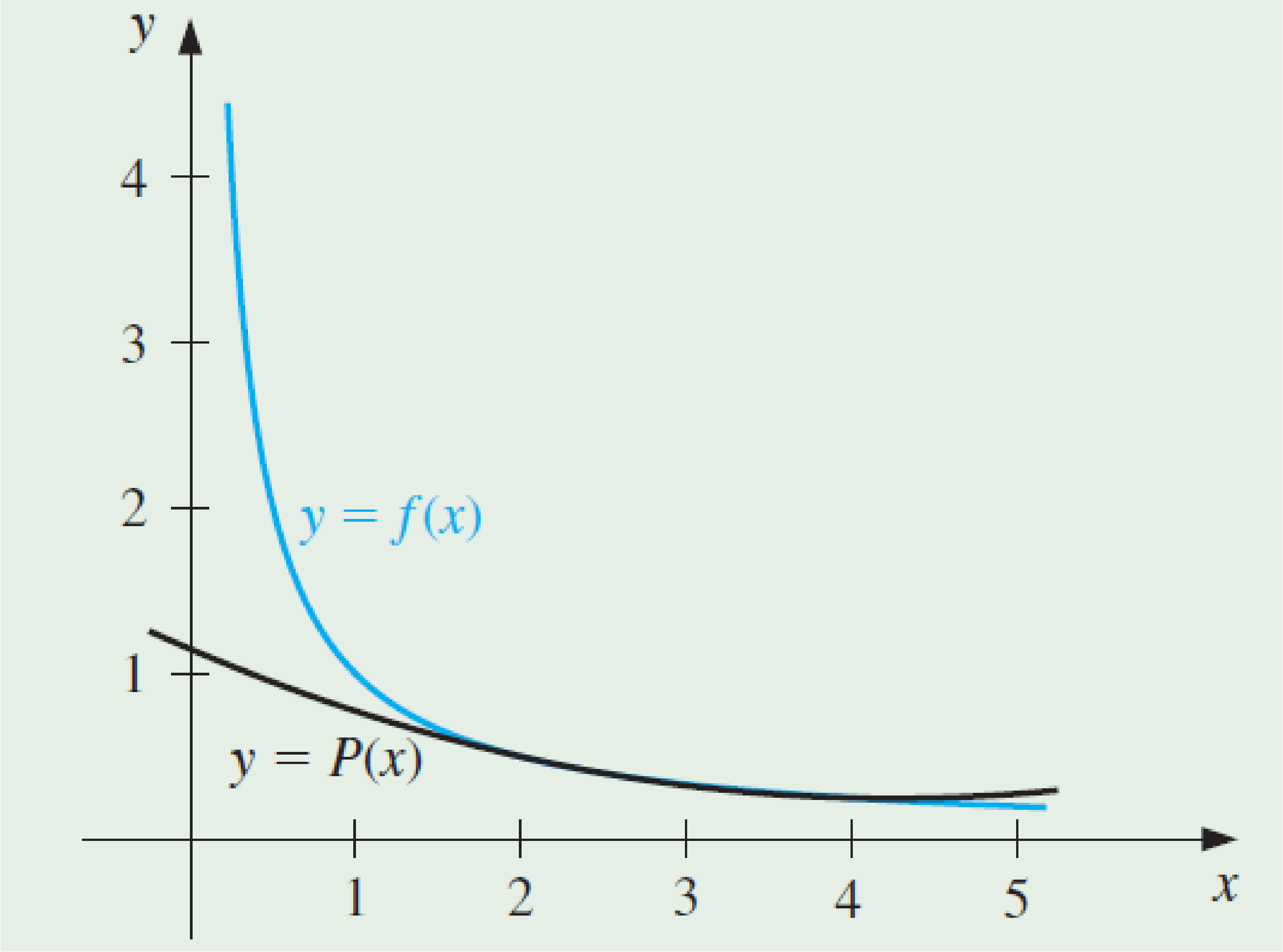
Part (b): Solution

- An approximation to $f(3) = 1/3$ is
- $f(3) \approx P(3) = (9/22) - (105/88) + (49/44)$

$$=(29/88)$$

$$\approx 0.32955.$$

The Lagrange Polynomial: Degree n Construction ...cont



Problems on Lagrange's Interpolation

1. The velocity distribution of a fluid near a flat surface is given below

x	0.1	0.3	0.6	0.8
V	0.72	1.81	2.73	3.47

Where x is the distance from the surface (mm) and V is the velocity (mm/sec). Use Lagrange's interpolation polynomial

Problems on Lagrange’s Interpolation

2. Using the following points, fit a polynomial using Lagrange’s method and find the value of y at x=2.7

x	2.10	2.5	3.10	3.50
y	5.14	6.78	10.29	13.58

Problems on Lagrange's Interpolation

3. Find the polynomial $f(x)$ by using Lagrange's interpolation formula and hence fit $f(3)$ for the following series

x	0	1	2	5
y	2	3	12	147

Problems on Lagrange's Interpolation

4. Using Lagrange's formula, find a unique polynomial $P(x)$ of degree 2 or less and hence evaluate $P(1.5)$

x	1	3	4
y	1	27	54

Problems on Lagrange's Interpolation

5. Using suitable interpolation formula find a polynomial which passes the points

x	0	1	3	4
y	-12	0	6	12

Problems on Lagrange's Interpolation

6. Use Lagrange's method to fit a polynomial to the following points and find the value of y at x=1

x	-1	0	2	3
y	-8	3	1	12

Reference Books

1. Steven C. Chapra, Raymond P. Canale, Numerical Methods for Engineers, 4/e, Tata McGraw Hill Editions

2. Dr. B. S. Garewal, Numerical Methods in Engineering and Science, Khanna Publishers,.

3. Steven C. Chapra, Applied Numerical Methods with MATLAB for Engineers and Scientist, Tata Mc-Graw Hill Publishing Co-Ltd

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Thank You

Newton’s Forward and Backward Difference Interpolation

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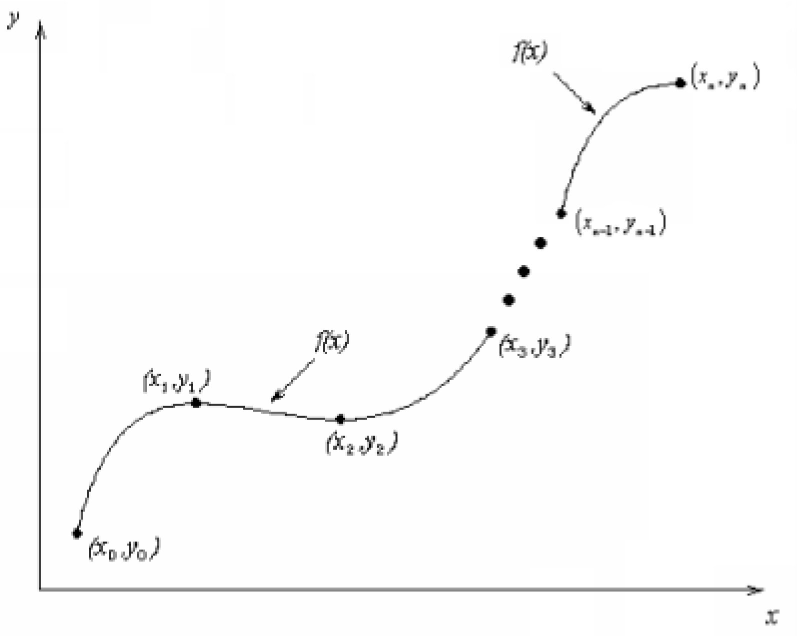
What is Interpolation ?

- Given (x_0,y_0), (x_1,y_1), …… (x_n,y_n), find the value of 'y' at a value of 'x' that is not given.



Lagrange Interpolation Disadvantages

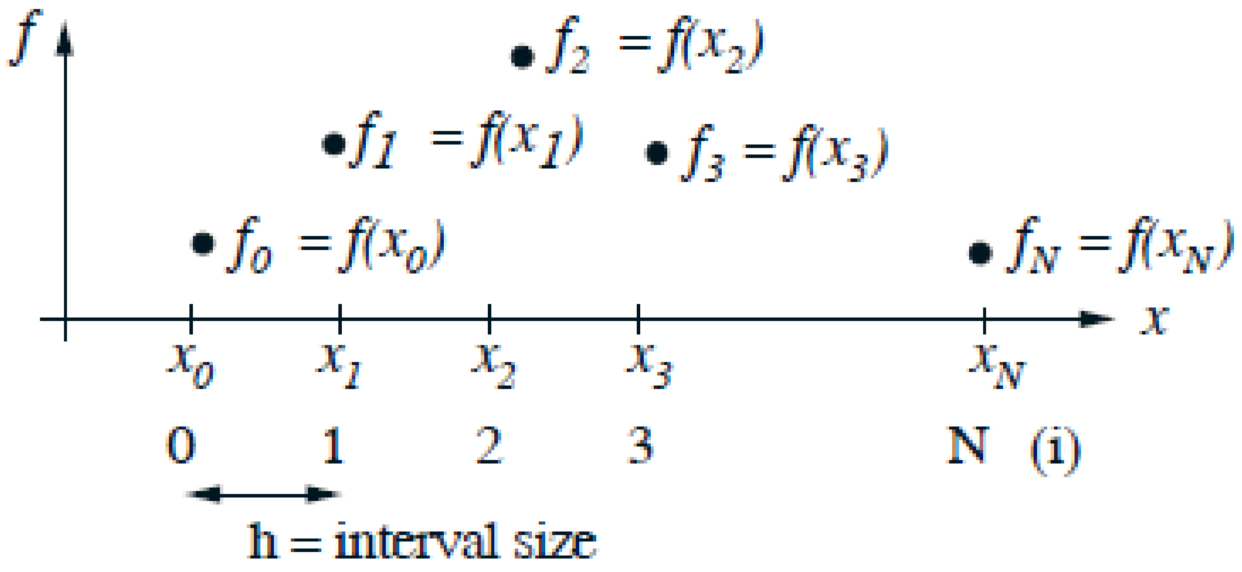
- The amount of computation required is large
- Interpolation for additional values of requires the same amount of effort as the first value (i.e. no part of the previous calculation can be used)
- When the number of interpolation points are changed (increased/decreased), the results of the previous computations can not be used
- Error estimation is difficult (at least may not be convenient)

Newton's Forward Interpolation

- Use Newton Interpolation which is based on developing difference tables for a given set of data points
- The N^{th} degree interpolating polynomial obtained by fitting $N + 1$ data points will be identical to that obtained using Lagrange formulae!
- Newton interpolation is simply ***another*** technique for obtaining the same interpolating polynomial as was obtained using the Lagrange formulae

Newton's Forward Interpolation ...cont

- We assume equi-spaced points



Newton's Forward Interpolation ...cont

- Forward differences are now defined as follows:

- $\Delta^0 f_i = f_i$ (Zeroth order forward difference)
- $\Delta f_i = f_{i+1} - f_i$ (First order forward difference)
- $\Delta^2 f_i = \Delta f_{i+1} - \Delta f_i$ (Second order forward difference)

$$\Delta^2 f_i = (f_{i+2} - f_{i+1}) - (f_{i+1} - f_i)$$

$$\Delta^2 f_i = f_{i+2} - 2f_{i+1} + f_i$$

Newton's Forward Interpolation ...cont

- $\Delta^3 f_i = \Delta^2 f_{i+1} - \Delta^2 f_i$ (Third order forward difference)

  $\Delta^3 f_i = (f_{i+3} - 2f_{i+2} + f_{i+1}) - (f_{i+2} - 2f_{i+1} + f_i)$

  $\Delta^3 f_i = f_{i+3} - 3f_{i+2} + 3f_{i+1} + f_i$

- $\Delta^k f_i = \Delta^{k-1} f_{i+1} - \Delta^{k-1} f_i$ (k^{th} order forward difference)

Newton's Forward Interpolation ...cont

- Typically we set up a difference table

i	f_i	Δf_i	$\Delta^2 f_i$	$\Delta^3 f_i$	$\Delta^4 f_i$
0	f_o	$\Delta f_o = f_1 - f_o$	$\Delta^2 f_o = \Delta f_1 - \Delta f_o$	$\Delta^3 f_o = \Delta^2 f_1 - \Delta^2 f_o$	$\Delta^4 f_o = \Delta^3 f_1 - \Delta^3 f_o$
1	f_1	$\Delta f_1 = f_2 - f_1$	$\Delta^2 f_1 = \Delta f_2 - \Delta f_1$	$\Delta^3 f_1 = \Delta^2 f_2 - \Delta^2 f_1$	
2	f_2	$\Delta f_2 = f_3 - f_2$	$\Delta^2 f_2 = \Delta f_3 - \Delta f_2$		
3	f_3	$\Delta f_3 = f_4 - f_3$			
4	f_4				

Newton's Forward Interpolation ...cont

- Note that to compute higher order differences in the tables, we take forward differences of previous order differences instead of using expanded formulae.
- The order of the differences that can be computed depends on how many total data points, $x_0 \ldots\ldots\ldots x_N$, are available
- N + 1 ***data points can develop up to Nth order forward differences***

Newton's Forward Interpolation ...cont

- Then put the values in the formula
- $f_g = f_0 + u\,\Delta f_o + ((u(u\text{-}1)) / 2\,!)\,\Delta^2 f_o + ((u(u\text{-}1)\,(u\text{-}2)) / 3\,!)\,\Delta^3 f_o + ((u(u\text{-}1)\,(u\text{-}2)\,(u\text{-}3)) / 4\,!)\,\Delta^4 f_o$ ....

Where $u = ((x_g - x_0) / h)$

h = step size

Newton’s Forward Interpolation ...cont

Example: From the following table of yearly premium for policies maturity at coming ages, estimate the premiums for policies maturity at the age of 46 years.

Age (x)	45	50	55	60	65
Premium (y)	2.871	2.404	2.083	1.862	1.712

Newton's Forward Interpolation ...cont

Solution: (1) Calculate the value of h and u

$h = x_1 - x_0$

$= 55 - 45$

$= 5$

$u = ((x_g - x_0) / h)$

$= ((46 - 45) / 5)$

$= 0.2$

Newton's Forward Interpolation ...cont

(2) Prepare the Newton's Forward difference table

x	y	Δy_o	$\Delta^2 y_o$	$\Delta^3 y_o$	$\Delta^4 y_o$
45	2.871	-0.467	0.146	-0.046	0.017
50	2.404	-0.321	0.1	-0.029	
55	2.083	-0.221	0.071		
60	1.862	-0.15			
65	1.712				

Newton's Forward Interpolation ...cont

(3) Calculate the value of y_g at $x_g = 46$

$$y_g = y_0 + u\,\Delta y_o + ((u(u-1)) / 2\,!)\,\Delta^2 y_o + ((u(u-1)\,(u-2)) / 3\,!)\,\Delta^3 y_o + ((u(u-1)\,(u-2)\,(u-3)) / 4\,!)\,\Delta^4 y_o$$

$$y_g = 2.871 + (0.2)(-0.467) + ((0.2(0.2 -1)) / 2\,!)\,(0.146) + ((0.2(0.2 -1)\,(0.2 -2)) / 3\,!)\,(-0.046) + ((0.2(0.2 -1)\,(0.2 -2)\,(0.2 -3)) / 4\,!)\,(0.017)$$

Newton's Forward Interpolation ...cont

- $y_g = 2.871 - 0.0934 - 0.01168 - 2.208 \times 10^{-3} - 5.712 \times 10^{-4}$

- $y_g = 2.7631$

- At $x_g = 46$, $y_g = 2.7631$ (Answer)

Problems on Newton's Forward Interpolation

1. State the order of polynomial which might be suitable for following function. Calculate f(3.5) using forward difference formula

x	2	3	4	5	6	7	8	9
y	19	48	99	178	291	444	643	894

Problems on Newton's Forward Interpolation

2. Find the value of y for x=0.5 for the following table of x, y values using Newton's forward difference formula

x	0	1	2	3	4
y	1	5	25	100	250

Problems on Newton's Forward Interpolation

3. From the tabulated values of x and y given below prepare forward difference table. Find the polynomial passing through the points and estimate the value of y when $x=1.5$. Also find the slope of curve at $x=1.5$

x	0	2	4	6	8
y	5	29	125	341	725

Problems on Newton's Forward Interpolation

4. Find the polynomial passing through the following points using Newton's forward difference formula and hence find y and dy/dx at x =0.5

x	0	1	2	3	4	5
y	1	1	7	25	61	12

Problems on Newton's Forward Interpolation

5. Following is the table of square roots. Calculate the values of square root of 151 and 155 by using Newton's forward difference formula

x	0	1	3	4
y	-12	0	6	12

Problems on Newton's Forward Interpolation

6. The following data are taken from the steam table. Find the pressure at $t=142^{0}C$ using Newton's forward difference formula

T (^{0}C)	140	150	160	170	180
P (Kgf/cm^2)	3.685	4.854	6.302	8.076	10.225

Problems on Newton's Forward Interpolation

7. The velocity distribution of a fluid near a flat surface is given below. Where x is the distance from the surface (cm) and V is the velocity (cm/sec). Using Newton's forward interpolation formula obtain the velocity at x=0.4, 0.4, 0.6 & 0.8

x	0.1	0.3	0.5	0.7	0.9
V	0.72	1.81	2.73	3.47	3.98

Newton’s Backward Interpolation

- Newton backward interpolation is essentially the same as Newton forward interpolation except that backward differences are used

Newton's Backward Interpolation ...cont

- Backward differences are now defined as follows:

- $\nabla^0 y_i = y_i$ (Zeroth order forward difference)
- $\nabla y_i = y_i - y_{i-1}$ (First order forward difference)
- $\nabla^2 y_i = \nabla y_i - \nabla y_{i-1}$ (Second order forward difference)
- $\nabla^k y_i = \nabla^{k-1} y_i - \nabla^{k-1} y_{i-1}$ (k^{th} order forward difference)

Newton's Backward Interpolation ...cont

- Typically we set up a difference table

x	y	∇ y	∇^2y	∇^3y
x_0	y_0			
x_1	y_1	$\nabla y_1 = y_1 - y_0$		
x_2	y_2	$\nabla y_2 = y_2 - y_1$	$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$	
x_3	y_3	$\nabla y_3 = y_3 - y_2$	$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$	$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$

Newton's Backward Interpolation …cont

- Then put the values in the formula
- $y_g = y_{n-1} - u \nabla y_{n-1} + ((u(u-1)) / 2!) \nabla^2 y_{n-1} - ((u(u-1)(u-2)) / 3!) \nabla^3 y_{n-1} + ((u(u-1)(u-2)(u-3)) / 4!) \nabla^4 y_{n-1}$ ……………………

Where $u = ((x_{n-1} - x_g) / h)$

h = step size

Newton's Backward Interpolation ...cont

Example: For the following data using backward difference polynomial interpolate at x = 0.25

x	0.1	0.2	0.3	0.4	0.5
y	1.4	1.56	1.76	2.00	2.28

Newton’s Backward Interpolation ...cont

Solution: (1) Calculate the value of h and u

$h = x_1 - x_0$

$= 0.2 - 0.1$

$= 0.1$

$u = ((x_{n-1} - x_g) / h)$

$= ((0.5 - 0.25) / 0.1)$

$= 1.5$

Newton's Backward Interpolation ...cont

(2) Prepare the Newton's backward difference table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
0.1	1.4				
0.2	1.56	0.16			
0.3	1.76	0.2	0.04		
0.4	2.0	0.24	0.04	0	
0.5	2.28	0.28	0.04	0	0
----	y_{n-1}	∇y_{n-1}	$\nabla^2 y_{n-1}$	$\nabla^3 y_{n-1}$	$\nabla^4 y_{n-1}$

Newton's Backward Interpolation ...cont

(3) Calculate the value of y_g at $x_g = 0.25$

- $y_g = y_{n-1} - u \nabla y_{n-1} + ((u(u-1)) / 2!) \nabla^2 y_{n-1} - ((u(u-1)(u-2)) / 3!) \nabla^3 y_{n-1} + ((u(u-1)(u-2)(u-3)) / 4!) \nabla^4 y_{n-1}$
- $y_g = 2.28 - (1.5)(0.28) + ((1.5(1.5-1)) / 2!)(0.04) - 0 + 0$
- $y_g = 1.655$
- At $x_g = 0.25$, $y_g = 1.655$ (Answer)

Problem on Newton's Backward Interpolation

1. Prepare the backward difference table for the given values of x and y

x	0	2	4	6	8	10
y	2	6	8	9	11	15

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8. S. S. Sastry, Introductory Methods of Numerical Analysis, PHI.

Thank You